Reply by Authors to J. G. Simmonds

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ADDITIONAL evidence of the accuracy and convergence associated with a recently developed iterative technique[1, 2] was presented in Ref. 3. This was accomplished by comparing the iterative solution for a uniformly loaded annular membrane, fixed at the outer boundary and free of tractions and support at the inner boundary with a power series solution.

In the preceding comment Simmonds draws attention to that portion of the paper dealing with the power series solution. The alternate procedure employed by Simmonds leads precisely to the solution obtained by the authors. Indeed, making use of the transformation

$$\Phi = (1 - \lambda^2)/a_1^3 \qquad \eta = 2 - (1 + \nu)(1 - \lambda^2) \tag{1}$$

in the controlling algebraic equation for a_1, λ, and ν [Eqs. (24b), Ref. 4] leads directly to the relations

$$1.3a_1^3 = (1 + \nu)A_1^3 \qquad (1 - \lambda^2)/a_1^3 = (1 - \Lambda^2)/A_1^3 \tag{2}$$

given by Simmonds. Conversely, the governing equation for Φ [Eq. (10) of the preceding comment] can be recast to permit a direct solution for a priori specified values of ν and λ.

Invariant relations such as Eqs. (2) increase the usefulness of the available numerical results with a minimal amount of additional calculations. However, Simmonds remarks are somewhat optimistic. The results presented in Table 1 of Ref. 3 are insufficient to provide a complete spectrum of coefficients. To take full advantage of the invariant relations requires a complete table of results based on $\nu = 0.5$. Therefore, to augment the results given in Ref. 3, a more complete Table 1 (for $\nu = 0.5$) is given. The necessary equations to be used in conjunction with this table are

$$\Lambda = \left[1 - \frac{1.5}{1 + \nu}(1 - \lambda^2)\right]^{1/2} \qquad A_1 = \left(\frac{1.5}{1 + \nu}\right)^{1/3} a_1 \tag{3}$$

which replace Eqs. (19) of the preceding note.

Table 1 Coefficients a_1 for $\nu = 0.5$ and various values of λ

λ	a_1	λ	a_1
0	1.845	0.36	1.649
0.01	1.845	0.37	1.639
0.02	1.844	0.38	1.630
0.03	1.844	0.39	1.620
0.04	1.842	0.40	1.610
0.05	1.841	0.41	1.600
0.06	1.839	0.42	1.590
0.07	1.836	0.43	1.580
0.08	1.834	0.44	1.570
0.09	1.831	0.45	1.559
0.10	1.827	0.46	1.548
0.11	1.824	0.47	1.538
0.12	1.820	0.48	1.527
0.13	1.816	0.49	1.516
0.14	1.811	0.50	1.505
0.15	1.806	0.51	1.494
0.16	1.801	0.52	1.483
0.17	1.795	0.53	1.471
0.18	1.790	0.54	1.460
0.19	1.784	0.55	1.448
0.20	1.777	0.56	1.436
0.21	1.771	0.57	1.424
0.22	1.764	0.58	1.412
0.23	1.757	0.59	1.400
0.24	1.750	0.60	1.388
0.25	1.743	0.61	1.375
0.26	1.735	0.62	1.362
0.27	1.727	0.63	1.350
0.28	1.719	0.64	1.336
0.29	1.711	0.65	1.323
0.30	1.703	0.66	1.310
0.31	1.694	0.67	1.296
0.32	1.685	0.68	1.282
0.33	1.676	0.69	1.268
0.34	1.667	0.70	1.254
0.35	1.658		

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Distribution of Nearly Circular Orbits

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1. Introduction

IN a recent paper[1] the theory of errors was employed to assess the precision of a guidance system designed to place a payload in a prescribed circular orbit. The essence of the analysis consists in determining the distribution of the square of the orbital eccentricity

$$e^2 = f(\beta, r, v) = \sin^2\beta + [(rv^2/gR^2) - 1]^2 \cos^2\beta \tag{1}$$

based on assumed normal distributions of the orbital parameters β, r, v at burnout. The distance r is measured from the center of the earth, v is the velocity, β is the heading angle measured outward from the normal to the radius vector, and g and R are the gravitational constant and earth radius.

In the referenced paper, the solution is obtained, approximately, first in terms of the gamma distribution and then,

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